

European Girls' Mathematical Olympiad, 2012

April 10th – 16th 2012

Cambridge, UK

UK Leader's Report

Introduction

This Olympiad was a brand new competition, designed to increase the participation of girls in international maths competitions. At the IMO normally around 10% of the participants are female: 57/564 in 2011 and 47/517 in 2010. The UK were invited to send a team to the China Girls' Maths Olympiad in 2010. The Chinese had started the competition in 2002 after organisers noticed that very few girls were selected for the national team. Encouraged by the success of that competition, the UK decided to start a similar competition in Europe. 19 countries participated in EGMO, including three guest countries: the USA, Saudi Arabia and Indonesia. Next year's hosts will be Luxembourg. More information on the competition can be found at:

<http://www.egmo2012.org.uk/>

In order to prepare for the competition, a talent search was launched in 2011 to introduce more girls to and involve more girls in olympiad mathematics. After we wrote to schools and publicised the talent search through various outlets, the girls interested in working towards selection for the UK team sat the first UK Mathematical Olympiad for Girls (UK MOG) in June. Eight girls were then invited to a September training camp at Queen's College, Oxford. This camp has traditionally marked the start of the olympiad training calendar. It is for students who have never previously attended any olympiad training camps and aims to introduce them to the kinds of mathematics that feature in olympiads. Further training included participation in the Advanced Mentoring Scheme and training camps in Hungary and Trinity College, Cambridge, as part of the UK IMO effort.

The selection for the EGMO team was made from performances at BMO1 and BMO2, the two rounds of the British Mathematical Olympiad. It was heartening to see many good performances in both competitions, and selection was a close-run affair.

The team consisted of:

UNK1	Natalie Behague	Dartford Grammar School for Girls
UNK2	Elizabeth Lee	Loughborough High School
UNK3	Ella Mi	The King's School, Peterborough
UNK4	Katya Richards	School of St Helen and St Katharine

The reserve was Emily Bain of Durham Johnston School. The leader was Alison Zhu of Trinity College, Cambridge and the deputy leader was Jo Harbour of Wolvercote Primary School. Katya has also been selected for the IMO9, from which the UK team for the IMO in Argentina this year will be chosen.

The competition

The format of EGMO was similar to that of other international competitions. Contestants sat two four-and-a-half hour papers on consecutive days, with four questions on each paper. As at the IMO, each question was marked out of 7, with part marks only given for significant progress. Thus marks of 3 or 4 were rarely awarded.

The scores of the UK team were as follows:

	Day 1				Day 2				Total	
	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8		
Natalie Behague	7	1	0	0	3	7	5	0	23	Bronze medal
Elizabeth Lee	0	1	0	0	0	0	0	0	1	
Ella Mi	0	1	0	0	1	0	0	0	2	
Katya Richards	0	7	2	0	1	0	7	0	17	Bronze medal

This was a solid performance from the team. Natalie was unlucky not to get a silver, missing out due to a couple of slips. Amusingly, the team scored highest on Q7, the tricky geometry on Day 2, outperforming their score on Q1, the supposedly easier geometry question from the first day.

The medal cut-offs were 35 for gold, 26 for silver and 14 for bronze. Overall, the UK finished 12th out of 19 countries, a commendable result. The competition was won by Poland by the narrowest of margins, beating the Romanian team into second place by one mark.

The Questions

The questions for the two days are as below, along with the author of each question.

Day 1

Problem 1: Let ABC be a triangle with circumcentre O . The points D , E and F lie in the interiors of the sides BC , CA and AB respectively, such that DE is perpendicular to CO and DF is perpendicular to BO . (By *interior* we mean, for example, that the point D lies on the line BC and D is between B and C on that line.)

Let K be the circumcentre of triangle AFE . Prove that the lines DK and BC are perpendicular.

Merlijn Staps (Netherlands)

Problem 2: Let n be a positive integer. Find the greatest possible integer m , in terms of n , with the following property: a table with m rows and n columns can be filled with real numbers in such a manner that for any two different rows $[a_1, a_2, \dots, a_n]$ and $[b_1, b_2, \dots, b_n]$ the following holds:

$$\max(|a_1 - b_1|, |a_2 - b_2|, \dots, |a_n - b_n|) = 1.$$

Tomasz Kobos (Poland)

Problem 3: Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(yf(x+y) + f(x)) = 4x + 2yf(x+y)$$

for all $x, y \in \mathbb{R}$.

Birgit van Dalen (Netherlands)

Problem 4: A set A of integers is called *sum-full* if $A \subseteq A + A$, i.e. each element $a \in A$ is the sum of some pair of (not necessarily different) elements $b, c \in A$. A set A of integers is said to be *zero-sum-free* if 0 is the only integer that cannot be expressed as the sum of the elements of a finite nonempty subset of A .

Does there exist a sum-full zero-sum-free set of integers?

Dan Schwarz (Romania)

Day 2

Problem 5: The numbers p and q are prime and satisfy

$$\frac{p}{p+1} + \frac{q+1}{q} = \frac{2n}{n+2}$$

for some positive integer n . Find all possible values of $q - p$.

Pierre Haas (Luxembourg)

Problem 6: There are infinitely many people registered on the social network *Mug-book*. Some pairs of (different) users are registered as *friends*, but each person has only finitely many friends. Every user has at least one friend. (*Friendship is symmetric; that is, if A is a friend of B , then B is a friend of A .*)

Each person is required to designate one of their friends as their *best friend*. If A designates B as her best friend, then (unfortunately) it does not follow that B necessarily designates A as her best friend. Someone designated as a best friend is called a *1-best friend*. More generally, if $n > 1$ is a positive integer, then a user is an *n -best friend* provided that they have been designated the best friend of someone who is an *$(n - 1)$ -best friend*. Someone who is a *k -best friend* for every positive integer k is called *popular*.

- (a) Prove that every popular person is the best friend of a popular person.
- (b) Show that if people can have infinitely many friends, then it is possible that a popular person is not the best friend of a popular person.

Dan Schwarz (Romania)

Problem 7: Let ABC be an acute-angled triangle with circumcircle Γ and orthocentre H . Let K be a point of Γ on the other side of BC from A . Let L be the reflection of K in the line AB , and let M be the reflection of K in the line BC . Let E be the second point of intersection of Γ with the circumcircle of triangle BLM . Show that the lines KH , EM and BC are concurrent. (*The orthocentre of a triangle is the point on all three of its altitudes.*)

Pierre Haas (Luxembourg)

Problem 8: A *word* is a finite sequence of letters from some alphabet. A word is *repetitive* if it is a concatenation of at least two identical subwords (for example,

$ababab$ and $abcabc$ are repetitive, but $ababa$ and $aabb$ are not). Prove that if a word has the property that swapping any two adjacent letters makes the word repetitive, then all its letters are identical. (Note that one may swap two adjacent identical letters, leaving a word unchanged.)

Dan Schwarz (Romania)

The questions were both interesting and challenging. Questions 4 and 8 were found to be especially difficult, no students managing to complete a solution under exam conditions.

Leader's Diary

What follows is an account of events at EGMO from my perspective. Having competed in a few Olympiads as a student, this was my first time participating in a competition as a team leader.

Tuesday 10th April

I make the long and arduous journey up the hill to Murray Edwards College from Trinity. Today, most of the teams will be arriving in Cambridge and I spot a few of them heading into town armed with lanyards and a guide. At the college, I am met by members of the EGMO committee: Ceri Fiddes, Vicky Neale, Bev Deteouf and Joseph Myers, who are very busy in the EGMO office keeping a track of the arrival arrangements of all the teams. I am handed a flag, the very fetching team T-shirts, a crate of UKMT flasks and my EGMO backpack.

After locating my room, I go in search of my students. I soon find Katya, Natalie and Lizzy. Natalie has attempted the journey from the train station to Murray Edwards College by public transport; I am impressed by her audacity. Unfortunately she has failed to get off at the right stop and has seen more of the outskirts of Cambridge than anticipated. Emily follows soon after, with Ella joining us in time for dinner.

As the competition is being held in the UK, we are able to invite our reserve, Emily, to join the team for the competition. Having slipped through the grasps of EGMO training, she turned out two excellent performances in BMO1 and BMO2, and attacked the pre-EGMO correspondence course with great zeal. Emily's pink hair invites many remarks from students and leaders, and goes very well with our pink team T-shirts.

We are encouraged by the organisers to venture up to the dining hall, named 'The Dome', for obvious reasons. We strategically position ourselves beside the tea and

muffins, and are happily engaged with maths and cards until dinner.

In the evening, I am accosted by Ceri and Vicky who request an interview for the daily EGMO bulletin. The EGMO bulletins keep the students updated on what is going on and feature puzzles, reports and interviews. I happily oblige with a few words of wisdom.

Wednesday 11th April

This morning we have our first jury meeting. This is very exciting for me, as I have only seen the student side of before and feel privileged to be let into the discussion amongst ‘the elders’.

At international olympiads, it is customary to have a jury to make all the formal decisions at the competitions, which usually involves selecting the problems, agreeing to a mark scheme, and solving disputes from coordination where necessary. The jury consists of all the team leaders from each country. As the jury gets to see the papers before the students, the leaders are usually separated from the students before the competition days. Here, we operate an honour code and are forbidden from discussing any mathematics with the students from after the first jury meeting until the second day’s paper is over.

The jury is headed by Geoff Smith, formerly UK IMO team leader and now a member of the IMO Advisory board. He and Joseph Myers make up the Problem Selection Committee for the competition. Two draft papers have already been selected by the PSC, with a back-up question prepared for each selected question. We all peruse the problems, and Geoff asks for comments. One of the leaders remarks that the current Q4 has been seen before in a similar form; later on, it is found that the exact question has indeed appeared in an Italian selection test. Q2 is rejected on the grounds that it can be trivialised by a well-known theorem and is duly replaced.

We adjourn in time for the opening ceremony. A few (very) short speeches are made, followed by a whirlwind introduction of all the participating countries. Some countries have made an impressive effort with their uniforms: the Latvian, Ukrainian and Indonesian teams deserve a special mention for their coordinated outfits. Afterwards, we make our way outside for some team photographs. We find that the Polish leader’s T-shirt is a remarkably similar shade of pink to our team uniform.

The leaders spend the afternoon translating the two papers into the languages requested by the students. Of course, I have very little to do. Other leaders have more difficulty: the German version is produced by a collaboration of the mainly French-speaking Luxembourg and Swiss leaders.

In the evening, we check and agree to the translated versions. Unfortunately, the Ital-

ian leader has been stricken down by an unknown ailment and Alessandra Caraceni, the Italian deputy, steps in to fill his empty seat at the table.

Ceri and Natalie were called away in the afternoon for an interview with BBC Radio Cambridgeshire. They were met with questions like “You’re a bit of a clever clogs, then, are you?” and “Have you always been good at maths?”. Ceri and Natalie acquitted themselves very well, and the whole team enjoy listening to the interview on BBC iPlayer before retiring for an early night.

Thursday 12th April

Today the students sit the first exam paper. I meet my team at breakfast. They are very subdued, mentally preparing themselves for the exam. Their trusty guide Sally-Anne shepherds them to the Centre of Mathematical Sciences, where their exams will be taking place.

Meanwhile, the leaders settle down in the jury room, waiting for any questions from the students. In the first thirty minutes of the paper, if a student needs a definition of a word in a question, or needs a point in the question clarified, then they may ask a question to the jury. They must not be given any mathematical help though. Their question is faxed through to the jury: the respective leader takes the question and proposes an answer, which must then be agreed by the jury and finally signed off by the jury chair. This ensures that no students are unwittingly given extra help. Questions soon start to flood in. Most are about the set notation in Q4, which will have been unfamiliar to many of the students. Surprisingly, none of my students have asked about Q4, with two requesting the meaning of ‘max’ instead. The scene in the jury room is quite frantic; there are leaders simultaneously picking up questions, answering them, showing proposed answers to the jury, and getting Geoff’s autograph.

Before lunch, the coordinators pay us a visit. Each problem captain presents us with their proposed mark scheme and the jury offers comments. Some sensible suggestions are made, and the problem captains promise to mull it over during lunch.

While we eagerly await the students, Jo and I have a debate on which question we thought was easier: Q1 or Q2. I come down on the side of the geometry, whilst Jo believes that the combinatorics is easier. We have a chat about the exam over lunch. All the girls seem happy with how it went, with everyone claiming at least one solution. Natalie is particularly pleased as she managed to dash off a solution to Q1 in the last 10 minutes. Unfortunately there will be fatal flaws in three of the claimed solutions for Q2.

It is also Natalie’s 18th birthday today. She is presented with some cupcakes by the organisers and shares them with the team. Natalie’s parents have had the foresight to

arm her with a birthday cake, so there are many cake-related festivities throughout the day. (For more details, I would recommend the unofficial student's report.) Despite a forecast of heavy rain for the entire week, the rain has stayed away and the students are able to relax in the sunshine with the college cat Jake.

Later on in the afternoon, the scripts arrive from the exam site and Jo and I start marking. The time flies by and one of the students has to be sent to fetch us for dinner.

Friday 13th April

The morning of the second paper proceeds very much like the first, only the jury room is even more hectic. Despite there being only 70 students in total, 52 questions are asked. It seems that *Mugbook* and 'best friendship' is creating a lot of confusion. At any one point, most of the jury are queuing up to get a question signed off. Birgit, the Dutch leader, finds this experience very bizarre and records it photographically for posterity. The Americans and the Romanians emerge victorious from the Q&A sessions, with neither leader required to answer a question from any of their students. The mark scheme for the second day's paper is discussed and there is general agreement from the jury.

The students are now free and are taken punting around Cambridge to unwind. While they are gone, I do a double take as I pass Andrew Carlotti in the hall. He is an old-hand in the UK IMO squad, but does not have the required number of X-chromosomes to be competing in the competition. It turns out that he has an informatics camp in Cambridge and has arrived a little early to incorporate a detour to Murray Edwards College.

Later on, Jo and I pick up today's scripts; the marking is not quite as straightforward. There is a lot of writing on question 5 which will take us quite a while to decipher it. Katya's work on question 6 provides some light relief. She begins her write-up by remarking that "Best friendships are like a directed graph". Jo feels that this is a very appropriate description for children in her class. Eventually, we decide that we will have more success looking through Q5 in the morning, and so join the students for the Ceilidh in the Dome. Other leaders are there too – Pierre Haas, the Luxembourg leader, throws himself into the dancing with particular aplomb.

Saturday 14th April

Today we coordinate all the scripts. At olympiads, each script is read and marked twice: by the coordinators and by the leaders of the team. The coordinators will try to ensure that all the scripts are marked fairly and based on the same interpretation

of the mark scheme. The coordinators and leaders then meet to agree a fair mark for each student during the coordination phase. As the UK is hosting the competition, the coordinators are British. Therefore, we coordinate with the leaders of the country which set each question to ensure that the British scripts get fairly marked.

Joseph has created a new system where the scores are updated live, as soon as the marks are agreed during coordination. The students can go to the Long Room and follow their scores on the big screen, whilst family and friends can sit at home pressing the refresh button. This is an attempt to turn olympiad mathematics into a more exciting spectator sport.

Most of the questions involve very quick coordinations, especially Q4 and Q8 where we, like many other countries, collect our 8 zeros. Q5 takes rather longer. The consultation between the Luxembourg leadership and coordinators take so long that our slot is over before we can even start coordinating. This looks ominous. We have found some holes in Natalie's script for Q5, but they can both be patched up by fairly quick fixes. The Luxembourg leader is extremely suspicious as the word "prime" is not mentioned in either her neat or rough work. Eventually, after some discussion, we agree that the holes occur in two of the more significant parts of the solution, so her solution must go in the 0+ mark scheme; we settle for 3 and feel that this is fair. Ella's script also causes some trouble. She has tried an approach which involves looking at the denominator and numerator with common factors, which doesn't really work, but has actually proved some useful things along the way. She gets her 1. Question 7 also provokes some discussion. Ella has drawn a diagram in her rough work, and has decided to write down a indiscriminating list of everything she has observed about the diagram. She has reflected H in BC to H' and seems to realise that this should lie on the same line as EM . However, as she has worked in rough, none of her observations have been substantiated by proof, and so after much discussion, she cannot be given a mark. Natalie has made a logical slip at the end of her geometry solution, which is disappointing. The beauty of her solution, though, is remarked upon by both the problem setter and the coordinators. After dinner, we coordinate the last of our questions, Q2. Although only Katya has a perfect solution, all the students pick up a mark for their efforts.

We finish coordination just in time for the start of a talk to be given by Dan Schwarz, the leader of the Romanian team. He lectures on how to think about combinatorics questions, with an interesting dissection of Geoff's famous windmill problem from IMO 2011. Competitions are a great place to meet students from many different countries, but they also have the potential to be an excellent place to learn about interesting maths too. I think this is a great idea and hope talks like this continue to be a feature of future EGMOs.

Afterwards, the final jury meeting of the competition takes place, which is usually quite dramatic as it is when the medal boundaries are set. There are rules as to who

should get a medal, but you can never be sure how rigidly the rules will be enforced. The jury chair Geoff Smith begins by congratulating the jury for their suggestions on the mark scheme, describing their contributions as “unusually intelligent”. He also remarks that the coordinators had reacted by being “unusually reasonable”. It seems that the good relations between the jury and the coordinators are a rather remarkable feature of EGMO 2012.

Geoff presents the cut-offs according to the 1:2:3 rule. Several leaders speak strongly in favour of more generous cut-offs and we end up with a set of cut-offs such that no student has missed out on a medal by a single mark. The jury also has an interesting conversation on the structure of the competition, and perhaps the Luxembourg organizers will take these suggestions into account in 2013.

Sunday 15th April

The students go on an excursion to Bletchley Park. I excuse myself to slip away to do some revision for some fairly imminent exams. James Cranch, the UK IMO leader and a coordinator at this competition, very enthusiastically takes over as UK leader for the trip. He causes confusion on the way back; engrossed by teaching the team some linear algebra, he boards the wrong bus. I happily get my students back and we have a quick debrief on the papers before dressing for dinner.

The gala dinner is a jovial occasion. I am seated next to the Latvian leader who is able to enlighten me on many things about Latvian culture and mathematics competitions. Speeches again are both lovely and short. The students collect their medals, which are square in design and reassuringly heavy. The ceremony concludes with a handover of the EGMO flag from the British team to the Luxembourg team. There is slight panic on stage as none of the UK team can find the EGMO flag, which is actually just draped on the table behind them. The Polish team are presented with a shiny plate as the winners of the competition.

Conversation flows late into the night, as the students, leaders, coordinators, staff and guides celebrate the end of a wonderful week.

Monday 16th April

Teams have been leaving since the early hours of the morning, and the UNKs get up early to say their farewells. One by one, the team are picked up by their parents, leaving Natalie and Emily hanging around until the bitter end. Eventually, they decide that staying in Murray Edwards College with no-one else around might not be such a good idea, and trot off to catch trains. I return down the hill back to Trinity.

Closing Remarks

This competition has been a great experience owing much to the hard work and dedication of many people.

Firstly, I'd like to thank the team for being a pleasure to be around, for their enthusiasm, and for all their hard work in the lead-up to the competition. The UK team guide, Sally-Anne, looked after the team extremely well. Congratulations to Ceri Fiddes and Vicky Neale for organising such a brilliant event for all involved. We must thank our sponsors for helping us to put on such an event, in particular the London Mathematical Society and Jane Street. I am very grateful to Bev Detoef and the UKMT office who have smoothed out all the organisational logistics throughout the year. There are many UKMT volunteers who have helped with camps, mentoring and other items of training to prepare the team; a special mention must go to those who gave up their time over the summer to help with the EGMO mentoring scheme. Geoff Smith and James Cranch have offered me excellent advice and support as I took on the role of UK team leader for the first time. Last but not least, Jo has been lovely company and a great help, particularly as a sounding board to correct me when I am thinking something completely stupid!

Having experienced both the China Girls' Maths Olympiad and EGMO, I feel convinced that an all-girls competition is a worthwhile pursuit. The questions were not easy, and such a competition gives girls a challenging target to aim for, potentially as a stepping stone to success at the IMO. I hope that it will be effectual in its goal of raising the number of girls representing their countries at other international competitions. As remarked by the competition director, Ceri Fiddes, we will know that EGMO has been a success when there is no longer a need for such a competition.

Alison Zhu

Trinity College, Cambridge

May 2012